5 Algebraic skills and techniques

Syllabus reference

Algebraic modelling 3

- Algebraic skills and techniques

In this chapter algebraic techniques related to scientific notation, manipulation of algebraic expressions and methods of solving equations will be investigated.

ARE YOU READY?

Try the questions below. If you have difficulty with any of them, extra help can be obtained by completing the matching SkillSHEET. Either click on the SkillSHEET icon next to the question on the *Maths Quest HSC Course* eBookPLUS icon or ask your teacher for a copy.

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Scientific notation

Changing the subject of a formula

1 Write each of the following in scientific notation.

a 25 000 **b** 236 000 000 **c** 400 000 **d** 26 000 000 000 000

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Substitution into a formula

Substitution into a formula

2 For each of the following formulas, find the value of the subject, given the variables.

a $I = Prn$ given that $P = 5750$, $r = 0.08$ and $n = 5$ (answer correct to 1 decimal place)

b $A = pab$ given that $a = 5.6$ and $b = 9.2$

c $A = \frac{h}{3}(d_f + 4d_m + d_l)$ given that $h = 60$, $d_f = 0$, $d_m = 32$ and $d_l = 28$

d $A = \frac{1}{2}ab \sin C$ given that $a = 23.4$, $b = 37.1$ and $C = 60°$ (answer correct to 1 decimal place)

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Simplifying like terms

Simplifying like terms

3 Simplify each of the following expressions.

a $r + r + r + r + r$ **b** $7m + 9m - 6m$ **c** $9x + 7 + 8 + 7x$

d $9a + 2b - 8a - 7b$ **e** $2x + 4y - 3x$ **f** $5m - 5n + 4m - 3n$

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Multiplication using indices

Multiplication using indices

4 Simplify each of the following expressions.

a $r^4 \times r^6$ **b** $6a^3 \times 3$ **c** $4p^5 \times 7p$

d $5q^2 \times 7q^4$ **e** $12m \times 4m^5$ **f** $3r^2s^5 \times 9rs^6$

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Division using indices

Division using indices

5 Simplify each of the following expressions.

a $\frac{d^6}{d^2}$ **b** $\frac{30m^4}{5}$ **c** $\frac{42x^6}{x}$

d $\frac{4q^8}{2q^3}$ **e** $\frac{56rs^6}{7rs}$ **f** $\frac{4k^3}{32k}$

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Raising a power to a power

Raising a power to a power

6 Simplify each of the following expressions.

a $(a^2)^4$ **b** $(4b)^3$ **c** $(2c^2)^4$

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Solving linear equations

Solving linear equations

7 Solve each of the following equations.

a $z - 42 = 76$ **b** $4y = 96$ **c** $6w - 9 = 69$

d $\frac{9v}{6} = 8$ **e** $6(t - 5) = 54$ **f** $20 + 2n = n + 54$

5A Scientific notation

Scientific notation is used to express very large or very small numbers in terms of a power of 10. It is particularly useful in branches of science such as astronomy, where large distances are measured, or in biology, where very small measurements of microbes are taken.

In the preliminary course, it was shown that numbers are written in scientific notation by rewriting the number with a decimal point after the first significant figure. This decimal is then multiplied by the appropriate power of 10. This power of 10 is found by counting the number of places that the decimal point has been moved. When moving the decimal point left, the power of 10 is positive; it is negative when moving the decimal point to the right.

WORKED EXAMPLE 1

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Worked example 1

Write each of the following in scientific notation.
a 8 000 000
b 13 400 000 000
c 0.000 034 51

THINK		WRITE
a 1	Move the decimal point after the first significant figure.	**a** $8\,000\,000 = 8 \times 10^{6}$
2	The decimal point has been moved 6 places left.	
b 1	Move the decimal point after the first significant figure.	**b** $13\,400\,000\,000 = 1.34 \times 10^{10}$
2	The decimal point has been moved 10 places left.	
c 1	Move the decimal point after the first significant figure.	**c** $0.000\,034\,51 = 3.451 \times 10^{-5}$
2	The decimal point has been moved 5 places right.	

Measurements are often required to be rounded off to a given number of significant figures.

WORKED EXAMPLE 2

Write each of the following measurements in scientific notation, correct to 3 significant figures.
a **97 856 472 124 km** **b** **0.000 000 124 117 23 mg**

THINK	WRITE
a 1 Move the decimal point after the first significant figure.	**a** 9.79×10^{10} km
2 The decimal point has been moved 10 places left.	
3 Round the decimal off after the third significant figure.	
b 1 Move the decimal point after the first significant figure.	**b** 1.24×10^{-7} mg
2 The decimal point has been moved 7 places left.	
3 Round the decimal off after the third significant figure.	

To change a number from scientific notation back to a decimal number, move the decimal point to the right if the power of 10 is positive. If the power of 10 is negative, move the decimal point to the left. Zeros will need to be added if there are insufficient decimal places.

WORKED EXAMPLE 3

Write each of the following as a decimal number.
a 3.85×10^8
b 8.654×10^6

THINK	WRITE
a Move the decimal point eight places to the right. You will need to add six zeros to do this.	**a** $3.85 \times 10^8 = 385\,000\,000$
b Move the decimal point six places to the left. You will need to add five zeros after the decimal point to do this.	**b** $8.654 \times 10^{-6} = 0.000\,008\,654$

REMEMBER

1. Scientific notation is used as a shorthand way of writing very large and very small numbers.
2. The decimal point is placed after the first significant figure, and then this decimal is multiplied by the appropriate power of 10.
3. The power of 10 is found as follows.
   - If the decimal point is moved left (for large numbers), the power of 10 is the number of places moved left.
   - If the decimal point is moved right (for small numbers), the power of 10 is negative and is the number of places moved right.
4. To write a number given in scientific notation as a decimal number, move the decimal point:
   (a) to the right for a positive power of 10
   (b) to the left for a negative power of 10.

EXERCISE

5A Scientific notation

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Scientific notation

1 WE1a Write each of the following in scientific notation.
a 90 000 **b** 20 000 000 000 **c** 700

2 WE1b Write each of the following in scientific notation.
a 1 458 000 **b** 23 650 000 000 000 **c** 2589

3 WE1c Write each of the following in scientific notation.
a 0.000 000 02 **b** 0.004 57 **c** 0.000 000 000 049 321

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Scientific notation

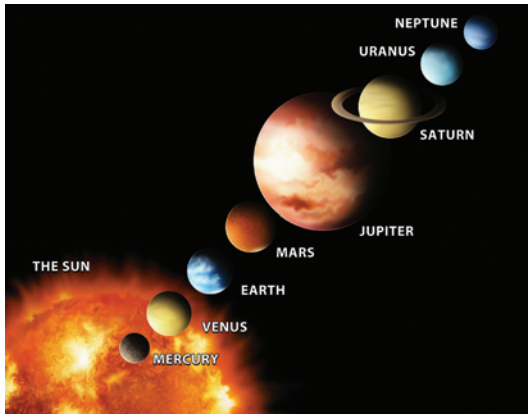
4 WE2 Write each of the following in scientific notation, correct to 3 significant figures.
a 93 154 789 km **b** 78 548 963 214 mm **c** 45 874 t
d 0.003 654 7 g **e** 0.213 658 mL **f** 0.000 005 687 4 s

5 WE3a Write each of the following as a decimal number.
a 3.4×10^4 **b** 2.87×10^6 **c** 3.0248×10^{10}

6 WE3b Write each of the following as a decimal number.
a 5.85×10^{-4} **b** 1.97×10^{-6} **c** 1.002×10^{-3}

7 An astronomical unit (AU) is defined to be the distance between the Earth and the sun and is equal to approximately 150 000 000 km. The table below shows the distance between each planet in the solar system and the sun in astronomical units. Write the distance between each planet and the sun in kilometres in scientific notation, correct to 3 significant figures.

Planet	Distance (AU)	Distance in km (scientific notation)
Mercury	0.39	
Venus	0.72	
Earth	1.0	
Mars	1.52	
Jupiter	5.20	
Saturn	9.54	
Uranus	19.18	
Neptune	30.06	



8 Complete each of the measurement conversions.
a 2.35×10^7 mm = ___ m **b** 8.4×10^7 m = ___ km
c 6.4×10^5 cm = ___ mm **d** 6.58×10^6 kg = ___ t
e 7.802×10^6 t = ___ kg **f** 8.29×10^{10} kg = ___ g
g 1.87×10^8 L = ___ kL **h** 2.178×10^7 kL = ___ L
i 5.55×10^7 L = ___ mL

Further development

9 Write each of the following in scientific notation and then simplify. Keep your answer in scientific notation.
a 45 000 × 34 000 **b** 230 × 450
c 78 000 × 6000 **d** 25 000 000 000 000 × 42 000 000 000 000 000
e 43.5 × 1200 **f** 780 694 000 000 × 45 000
g 90 700 × 450 000 **h** 67 925 000 × 320 000
i 34 × 80 000 000 000 **j** 1200 × 34 000
k 65 000 000 000 000 000 000 000 × 23 000 000 000 000 000 000
l 347 × 4500

10 Write each of the following in scientific notation and then simplify. Keep your answer in scientific notation.

a $4500 \div 50$
b $62\,310 \div 3000$
c $400\,000\,000 \div 20\,000$
d $480\,000 \div 1200$
e $78\,000\,000\,000\,000 \div 300\,000\,000$
f $5\,500\,000 \div 11\,000$
g $865\,000\,000\,000\,000\,000\,000\,000 \div 50\,000\,000\,000$
h $672\,312\,000 \div 3\,000\,000$
i $254 \div 20$
j $95\,000\,000 \div 5000$
k $12\,050 \div 500$
l $88\,000\,000\,000\,000\,000\,000\,000\,000 \div 1\,100\,000\,000\,000\,000$

11 MC

a What does $\dfrac{4.5 \times 10^4 \times 2.3 \times 10^7}{8.89 \times 10^8}$ equal?

A 1.164×10^3
B 1.164×10^4
C 1.164×10^{19}
D 11.64×10^3

b What does $\dfrac{1.02 \times 10^{12} \times 2.05 \times 10^8}{9.987 \times 10^7}$ equal?

A 20.937×10^{13}
B 2.0937×10^{14}
C 2.0937×10^{12}
D $0.209.37 \times 10^1$

12 If in a particular city there are 5.8×10^6 people and in the neighbouring city there are 4.2×10^6 people, how many people altogether live in the two cities?

13 There are approximately 2.2×10^7 people living in Australia at the moment. If in 20 years' time the population has grown to 3.1×10^7, by how much will the population have increased?

14 If Earth's orbit is about 9.38×10^8 km and Earth travels at an approximate speed of 1.06×10^5 km/h, how long (in hours) does it take for Earth to complete one orbit?

5B Substitution

Substitution was studied during the preliminary course. Substitution involves replacing a pronumeral in an expression with a numerical value. There are many different types of expressions that may need substitution.

A linear expression such as $3x + 5$ involves no index other than 1. When graphed, these expressions form a straight lines. When performing a substitution, we write the expression and the values of the known pronumerals, rewrite the expression having substituted the given values, and finally calculate the value of the expression.

WORKED EXAMPLE 4

The formula for the area of a trapezium is given by $A = \frac{h}{2}(a+b)$, where a and b are the parallel sides and h is the height. Find the area of a trapezium with parallel sides 4.2 cm and 7.9 cm and a height of 5.1 cm.

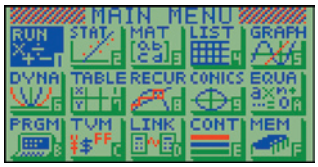
THINK	WRITE
Method 1: Technology-free	
1 Write down the given expression.	$A = \frac{h}{2}(a+b)$
2 Write down the variables where the values are known.	$a = 4.2, b = 7.9, h = 5.1$

3 Substitute the given values into the formula. $A = \frac{5.1}{2}(4.2 + 7.9)$

4 Evaluate. $A = 30.855$

Method 2: Technology-enabled

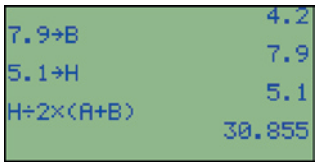
1 From the **MENU** select **RUN**.



2 Assign the value $a = 4.2$ by entering **4.2** ⟶ ALPHA **[A]** and then pressing EXE.

3 Repeat step 2 to assign the values $b = 7.9$ and $h = 5.1$.

4 Now enter the expression **H ÷ 2 × (A + B)** and then press EXE.



Many expressions involve higher powers or indices. An expression involving a power of 2 as its highest power is called a quadratic expression, an expression involving a power of 3 as its highest power is called a cubic expression.

Consider the expression $M = 6r^2$. In this expression, only the pronumeral r is raised to the power of 2. In the expression $M = (6r)^2$ the product $6 \times r$ is raised to the power of 2.

WORKED EXAMPLE 5

The expression $V = \frac{4}{3}\pi r^3$ is used to calculate the volume of a sphere. Find the volume of the sphere with a radius of 4.2 cm, giving your answer correct to 1 decimal place.

THINK	WRITE
1 Write down the given expression.	$V = \frac{4}{3}\pi r^3$
2 Write down the variables where the values are known.	$r = 4.2$
3 Substitute the given values into the formula.	$V = \frac{4}{3} \times \pi \times (4.2)^3$
4 Evaluate.	$V = 310.3 \text{ cm}^3$

Other expressions may involve taking square roots and cube roots. Care must be taken to use the calculator correctly. The square or cube root must be taken of the entire part of the expression that is under the root sign. This may involve using brackets.

WORKED EXAMPLE 6

The expression $r = \sqrt[3]{\frac{3V}{4\pi}}$ is used to calculate the radius of a sphere, given the volume. Find the radius of a sphere with a volume of 200 cm^3. (Give your answer correct to 1 decimal place.)

THINK	WRITE
1 Write down the given expression.	$r = \sqrt[3]{\frac{3V}{4\pi}}$
2 Write down the variables where the values are known.	$V = 200$
3 Substitute the given values into the formula.	$r = \sqrt[3]{\frac{3 \times 200}{4\pi}}$
4 Evaluate.	$r = 3.6$ cm

REMEMBER

1. Substitution involves replacing a pronumeral or pronumerals in an expression with numerical values.
2. Linear expressions involve only powers of 1.
3. Quadratic and cubic expressions involve powers of 2 and 3 respectively. In these expressions be sure to raise only the relevant part of the expression to the power.
4. Expressions that involve square and cube roots must be solved by correctly using a calculator and brackets.

EXERCISE

5B Substitution

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Changing the subject of a formula

1 WE4 In the formula $A = l \times b$, find the value of A, given that $l = 6.7$ and $b = 4.8$.

2 Find the value of each of the following by substituting into the formula.
- **a** $A = \frac{1}{2}bh$, if $b = 5$ and $h = 12.3$
- **b** $A = P + \frac{PRT}{100}$, if $P = 2000$, $R = 6.55$ and $T = 2.5$
- **c** $S = 2(lw + lh + wh)$, where $l = 3$, $w = 2.5$ and $h = 1.1$
- **d** $V = u + at$, where $u = 20$, $a = 4$ and $t = 25$
- **e** $T = a + (n - 1)d$, if $a = 66$, $n = 56$ and $d = {}^{-}4$

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Substitution

3 The formula $P = 2l + 2w$ is used to find the perimeter of a rectangle. Use the formula to find the perimeter of a rectangle, where $l = 3.5$ and $w = 9.7$.

4 The formula $C = \pi d$ is used to calculate the circumference of a circle. Use the formula to find the circumference of a circle with a diameter of 9.5 m. Give your answer correct to 1 decimal place.

5 In the formula $A = 6s^2$, find the value of A when $s = 5.5$.

6 WE5 Find the value of each of the following by substituting into the formula. (Give your answers correct to 2 decimal places.)
- **a** $V = r^2h$, if $r = 0.75$ and $h = 2.5$
- **b** $A = \pi(R^2 - r^2)$, if $R = 2.2$ and $r = 1$

c $V = \frac{4}{3}\pi r^3$, if $r = 3.2$

d $P = I^2R$, where $I = 0.6$ and $R = 230$

e $E = \frac{1}{2}mv^2$, where $m = 23$ and $v = 4.7$

7 The formula $A = 2\pi r^2 + 2\pi rh$ is used to calculate the surface area of a cylinder. Calculate the surface area of a cylinder with a radius of 2.3 cm and a height of 6.4 cm. Give your answer correct to the nearest whole number.

8 Use the formula $S = ut + \frac{1}{2}at^2$ to calculate the value of S, when $u = 9$, $t = 5$ and $a = 4.5$.

9 WE6 Use the formula $r = \sqrt{\frac{A}{4\pi}}$ to find the value of r (correct to 1 decimal place) when $A = 500$.

10 Use the formula $r = \sqrt[3]{\frac{3v}{4\pi}}$ to find the radius of a sphere with a volume of 1000 cm^3.

11 **a** Calculate the value of $S = \frac{a}{1-r}$, when $a = 8$ and $r = 0.2$.

b Calculate the value of $S = ut + \frac{1}{2}at^2$, when $u = 4.5$, $t = 6.1$ and $a = 4$.

c Calculate the value of $S = \sqrt{\frac{3V}{h}}$, when $V = 352.6$ and $h = 4.5$. (Give your answer correct to 1 decimal place.)

12 Find the value of each of the following by substitution into the formula. Where necessary, give your answer correct to 1 decimal place.

a $c = \sqrt{a^2 - b^2}$, when $a = 17$ and $b = 8$

b $T = 2\pi\sqrt{\frac{L}{g}}$, when $L = 65$ and $g = 9.8$

c $S = \sqrt{\frac{3V}{h}}$, if $V = 600$ and $h = 25$

d $r = \sqrt[3]{\frac{3V}{4\pi}}$, if $V = 900$

e $m = \sqrt{2xy} + y^3$, when $x = 2$ and $y = 3$

13 The formula $s = \sqrt{\frac{A}{6}}$ is used to calculate the side length of a cube having been given the area. Calculate the side length of a cube with a surface area of 162.24 cm^2.

Further development

14 The formula $A = \sqrt{s(s-a)(s-b)(s-c)}$ can be used to find the area of any triangle, where a, b and c are the side lengths and s is half the perimeter of the triangle. Given that the side lengths of a triangle are 4 cm, 8 cm and 9 cm:

a calculate the value of $s\left(s = \frac{a+b+c}{2}\right)$

b find the area of the triangle, correct to 1 decimal place.

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15 Use the formula $A = lb$ to find the value of A in scientific notation, correct to 3 significant figures, given that $l = 2.59 \times 10^4$ and $b = 3.92 \times 10^9$.

16 Earth's orbit around the sun is in the shape of an ellipse. The formula for finding the area of an ellipse is $A = \pi ab$. Given that $a = 9.3 \times 10^7$ km and $b = 9.2 \times 10^7$ km find the area enclosed by the Earth's orbit correct to 2 significant figures.

17 The formula $A = \frac{1}{2}ab \sin C$ is used to find the area of a triangle. Find the area of a triangle where $a = 2 \times 10^9$ mm, $b = 1.8 \times 10^6$ mm and $C = 75°$.
Give your answer in scientific notation correct to 3 significant figures.

18 Consider the formula $T = ar^{n-1}$. Find the value of T given that $a = 5.6$, $r = 10$ and $n = 12$. Give your answer in scientific notation.

19 Consider the formula $y = \frac{1}{x}$.

a Find the value of y when $x = 10$.

b Find the value of y when $x = \frac{1}{2}$.

c Explain what happens to the value of y as the value of x increases.

5C Algebraic manipulation

Basic manipulation of algebraic expressions was covered in the preliminary course. Algebraic expressions can be simplified by addition, subtraction, multiplication and division. Algebraic expressions are added and subtracted by collecting like terms. Only the same pronumeral or combination of pronumerals can be added together.

WORKED EXAMPLE 7

Simplify each of the following.

a $8x + 2x - 11x$ **b** $9a^2 + 2a + 4a^2 - 7a$

THINK	WRITE
a Each term uses the same pronumeral and so we add and subtract the coefficients.	**a** $8x + 2x - 11x = {}^{-}x$
b 1 Rewrite the expression, grouping the like terms together. Remember that a^2 and a are not like terms.	**b** $9a^2 + 2a + 4a^2 - 7a$
2 Complete each addition and subtraction separately.	$= (9a^2 + 4a^2) + (2a - 7a)$ $= 13a^2 - 5a$

To multiply and divide algebraic expressions, it is necessary to remember the index laws covered in the preliminary course.

First Index Law: $a^x \times a^y = a^{x+y}$

Second Index Law: $a^x \div a^y = a^{x-y}$ **or** $\frac{a^x}{a^y} = a^{x-y}$

Third Index Law: $(a^x)^y = a^{xy}$

When multiplying and dividing algebraic expressions it is important to remember to apply the index laws separately to each pronumeral.

WORKED EXAMPLE 8

Simplify each of the following fully.

a $6m^7 \times 7m^3$ **b** $\dfrac{48p^2q^4}{6pq^3}$ **c** $(5x^4)^3$

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Worked example 8

THINK	WRITE
a Multiply the coefficients and add the indices.	**a** $6m^7 \times 7m^3 = 42m^{10}$
b Divide the coefficients and subtract the indices for each pronumeral separately.	**b** $\dfrac{48p^2q^4}{6pq^3} = 8pq$
c Calculate 5^3 and multiply the indices.	**c** $(5x^4)^3 = 125x^{12}$

The manipulation of algebraic expressions also involves the expansion of brackets. When expanding brackets, we multiply every term inside the brackets by the term immediately outside the brackets.

WORKED EXAMPLE 9

Expand $2x^3(6xy - 9y^4)$.

THINK	WRITE
Multiply both terms inside the brackets by $2x^3$.	$2x^3(6xy - 9y^4) = 12x^4y - 18x^3y^4$

REMEMBER

1. Algebraic expressions are added and subtracted by collecting like terms.
2. Algebraic expressions are multiplied and divided using the index laws.
   - First Index Law: $a^x \times a^y = a^{x+y}$
   - Second Index Law: $a^x \div a^y = a^{x-y}$ or $\dfrac{a^x}{a^y} = a^{x-y}$
   - Third Index Law: $(a^x)^y = a^{xy}$
3. When using the index laws, apply each law to each pronumeral separately.
4. When expanding brackets, multiply every term inside the brackets by what is immediately outside.

EXERCISE
5C Algebraic manipulation

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Substitution into a formula

1 WE7a Simplify each of the following expressions.

a $4a + 8a$ **b** $14b + 4b$ **c** $23c - 9c$
d $35d + 6d$ **e** $5e - e$ **f** $16f - 15f$
g $g - 8g$ **h** $^-4h + 9h$ **i** $^-2i - 7i$
j $7j - 5j + 9j$ **k** $^-3k + 8k - k$ **l** $5l - 15l + 8l$

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Simplifying like terms

2 WE7b Simplify each of the following expressions.

a $8m + 3m - 9$ **b** $n - 4 + 7n$ **c** $7p^2 + 6p + 3p^2 - 2p$
d $5r + 9s - 2r + 2s$ **e** $7t + 1 - 4t - 7$ **f** $6u - 8v - 7u + 2v$
g $4w^2 - 3w^3 + 2w^2 - w^3$ **h** $2xy + 4xz - 3xy + xz$ **i** $4p^2 - 12 + p^2 - 4$
j $4x + 3y - 2xy + 6x - 4yx + y$

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Index laws

3 WE8a Simplify each of the following.

a $a^5 \times a^8$ **b** $b \times b^3$ **c** $3c^2 \times 4c^5$
d $d^3 \times 7d$ **e** $4p^4q^3 \times 3p^5q^2$ **f** $7gh \times 9g^2h^3$
g $4mn^4 \times 7m^5n$ **h** $4p^5 \times 5q^4$ **i** $6xyz \times 4x^2y^2$
j $6u^3v^3 \times 4v^2w^4 \times 2uw^5$

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Multiplication using indices

4 WE8b Simplify each of the following.

a $k^4 \div k$ **b** $15m^7 \div 5m^2$ **c** $48n^7 \div 8n^2$
d $\frac{14x^5}{7x}$ **e** $\frac{56m^4n^3}{7m^2n}$ **f** $\frac{45x^5}{9}$
g $m^6n^7 \div mn$ **h** $48p^3 \div 6q^3$ **i** $121a \div 11b$
j $32p^3q^4r^6 \div 4pqr^2 \div 2p^2q^2$

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Division using indices

5 WE8c Simplify each of the following.

a $(a^3)^4$ **b** $(2b^4)^2$ **c** $(3m^2)^3$
d $(4x^2y^3)^2$ **e** $(2pq^2)^4$

6 WE9 Expand each of the following.

a $2(m + 5)$ **b** $x(x + 2)$ **c** $3a(3a + 2b)$
d $3q^2(6q^4 - 2)$ **e** $5n(m - 5n)$ **f** $7a^2b^4(2a^4 - 3b^6)$
g $^-3(d + 5)$ **h** $^-3m(m - 2n)$ **i** $^-6r^3(2 - 3r^3)$
j $6pqr(3pq - r)$

7 Expand and simplify each of the following.

a $4(x + 2) + 2(3x - 1)$ **b** $a(a + 7) + 2(3a - 5)$ **c** $2m(m - n) + 6n(m - 2n)$
d $5(4x - 7) - 2(x + 5)$ **e** $2p(2p - 5) - 5(p - 6)$ **f** $2xy(3x - 4y) - x(y - xy)$

8 Fully simplify each of the following.

a $a^4 \times a^5 \div a^2$ **b** $(m^2)^3 \div m^4 \times m$ **c** $32m^2n^3 \div 4mn \times 2n^3$
d $4x^6y^7 \times 5xy^4 \div 2x^6$ **e** $(2z^3)^4 \div 8z^5 \div 2z^7$ **f** $\frac{9m^2n^4 \times 4mn^2}{(6mn^3)^2}$

Further development

9 Simplify each of the following.

a $[2(x - 5) - 3x] - 5x$
b $4m - [m(3m - 5) + m^2]$
c $7p - [5q - 5(q - p)] - 6q$

10 a MC Which of the following when expanded is equal to $10h + 12$?

A $2(5h + 6)$ **B** $10(h + 12)$ **C** $12(h + 10)$ **D** $5(2h + 6)$

b Which of the following when expanded is equal to $^-8 + 4r$?

A $^-4(-2 - r)$ **B** $^-4(2 - r)$ **C** $^-4(2 + r)$ **D** $^-4(2 - r)$

c Which of the following when expanded is equal to $8au - 10uh$?

A $2(4au - 5uh)$ **B** $8u(a - 1.25h)$ **C** $2u(4a + 5h)$ **D** $2u(4a - 5h)$

d Which of the following when expanded is equal to $30a^5n^4 + 12a^7n^3$?

A $6(5a^5n^4 + 2a^7n^3)$ **B** $6an(5a^4n^3 + 2a^6n^2)$
C $6a^5n^3(5an + 2a^2n)$ **D** $6a^5n^3(5n + 2a^2)$

11 Simplify each of the following.

a $\frac{x}{5} \times \frac{20}{y}$ **b** $\frac{x}{4} \times \frac{12}{y}$ **c** $\frac{y}{4} \times \frac{16}{x}$

d $\frac{x}{2} \times \frac{9}{2y}$ **e** $\frac{x}{10} \times \frac{25}{2y}$ **f** $\frac{3w}{14} \times \frac{7}{x}$

g $\frac{3y}{4x} \times \frac{8z}{7y}$ **h** $\frac{y}{3x} \times \frac{6z}{7y}$ **i** $\frac{x}{3z} \times \frac{9z}{2y}$

j $\frac{5y}{3x} \times \frac{x}{8y}$ **k** $\frac{20y}{7x} \times \frac{21z}{5y}$ **l** $\frac{y}{3w} \times \frac{x}{2y}$

12 Simplify each of the following.

a $\frac{15p^{12}}{5p^8}$ **b** $\frac{18r^6}{3r^2}$ **c** $\frac{45a^5}{5a^2}$

d $\frac{60b^7}{20b}$ **e** $\frac{100r^{10}}{5r^6}$ **f** $\frac{9q^2}{q}$

13 Simplify each of the following.

a $\frac{8p^6 \times 3p^4}{16p^5}$ **b** $\frac{12p^5 \times 4b^2}{18b^2}$ **c** $\frac{25m^{12} \times 4n^7}{15m^2 \times 8n}$

d $\frac{27x^9y^3}{12xy^2}$ **e** $\frac{16h^7k^4}{12h^6k}$ **f** $\frac{12j^8 \times 6f^5}{8j^3 \times 3f^2}$

g $\frac{8p^3 \times 7r^2 \times 2s}{6p \times 14r}$ **h** $\frac{27a^9 \times 18b^5 \times 4c^2}{18a^4 \times 12b^2 \times 2c}$ **i** $\frac{81f^{15} \times 25g^{12} \times 16h^{34}}{27f^9 \times 15g^{10} \times 12h^{30}}$

14 **a** $\left(\frac{3b^4}{d^3}\right)^2$ **b** $\left(\frac{5h^{10}}{2j^2}\right)^2$ **c** $\left(\frac{2k^5}{3t^8}\right)^3$

d $\left(\frac{7p^9}{8q^{22}}\right)^2$ **e** $\left(\frac{5y^7}{3z^{13}}\right)^3$ **f** $\left(\frac{4a^3}{7c^5}\right)^4$

5D Equations and formulas

The object of solving an equation is to find the value of an unknown pronumeral that makes that statement true. In solving an equation every process that has been performed on the pronumeral is reversed until it becomes the subject of the equation. In many cases, an equation arises as the result of substitution into a formula.

WORKED EXAMPLE 10

In the formula $C = 2\pi r$, find the value of r when $C = 100$, correct to 2 significant figures.

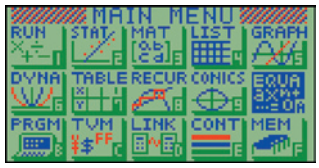
THINK	WRITE
Method 1: Technology-free	
1 Write the formula.	$C = 2\pi r$
2 Substitute the value of C.	$100 = 2\pi r$

3 Divide each side by 2π and round the answer off to 2 significant figures.

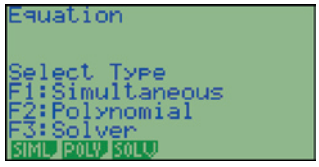
$$r = \frac{100}{2\pi}$$
$$= 16$$

Method 2: Technology-enabled

1 From the **MENU** select **EQUA**.



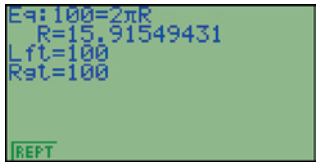
2 Press F3 **(Solver)**.



3 Delete any existing equation, and enter the equation that arises after the substitution is made. To enter $100 = 2\pi r$ press 1 0 0 SHIFT **[=] 2** SHIFT **[π]** ALPHA **[R]**.

Note: At this stage you may have a different value of R, but this is to be ignored.

4 Press F6 to solve the equation.



Some equations involve powers and roots. In the solution to an equation, remember that the opposite function to taking a square is to take the square root and vice versa. When solving such an equation, both the positive and negative square roots are possible solutions.

For example, the equation $x^2 = 9$ has the solution $x = \pm 3$. This differs from $\sqrt{9}$, which equals 3. When calculating the square root of a number, take the positive square root only.

WORKED EXAMPLE 11

In the equation $d = 5t^2$, find the value of t when $d = 320$.

THINK	WRITE
1 Write the formula.	$d = 5t^2$
2 Substitute the value of d.	$320 = 5t^2$
3 Divide each side by 5.	$t^2 = 64$
4 Take the square roots of each side, considering both the positive and negative answers.	$t = \pm 8$

Note: If the solver function on your graphics calculator is used only the positive solution is given. It is important to remain aware that equations of this type have a positive and negative solution.

With such examples, consider both the positive and negative cases only where appropriate. In practical cases where measurements are being considered, only the positive answer is given.

Using the same process as this, the subject of a formula can be changed. The subject of the formula is the single pronumeral usually written on the left-hand side of the formula. For example, in the formula $A = \pi r^2$, A is the subject. It is possible to make another pronumeral the subject of the equation by moving all other numbers and pronumerals to the other side of the formula, as if we were solving an equation.

WORKED EXAMPLE 12

Make x the subject of the formula $y = 5x - 2$.

THINK	WRITE
1 Write the equation.	$y = 5x - 2$
2 Add 2 to each side.	$y + 2 = 5x$
3 Divide each side by 5 (and write the new subject of the formula on the left-hand side).	$x = \frac{y+2}{5}$

This method is also used for quadratic formulas but, as with equation solving, it is important to remember to use both the positive and negative square root where appropriate.

WORKED EXAMPLE 13

The formula $A = 4\pi r^2$ is used to find the surface area of a sphere. Make r the subject of the formula.

THINK	WRITE
1 Write the formula.	$A = 4\pi r^2$
2 Divide both sides by 4π.	$\frac{A}{4\pi} = r^2$
3 Take the square root of each side. As r is the radius, a length, we consider only the positive square root.	$r = \sqrt{\frac{A}{4\pi}}$

REMEMBER

1. An equation can be formed after substitution into a formula.
2. When solving an equation, the object is to find the value of the unknown.
3. When an equation involves taking a square, the opposite function used to solve the equation is a square root.
4. Both the positive and negative square root should be taken unless the context of the equation means that only the positive should be used.
5. To make another pronumeral the subject of an equation, the same methods as for equation solving are used although we use pronumerals rather than make actual calculations.

EXERCISE

5D Equations and formulas

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SkillSHEET 5.7
doc-1356
Solving linear equations

1 WE10 The formula $C = \pi d$ is used to calculate the circumference of a circle. Find the diameter of a circle that has a circumference of 40 cm. Give your answer correct to 3 significant figures.

2 The formula $P = 2l + 2w$ is used to calculate the perimeter of a rectangle. Calculate the length of a rectangle that has a perimeter of 152 m and a width of 38 m.

3 In each of the following, find the value of the unknown after substitution into the formula. Where appropriate, give your answer correct to 1 decimal place.

a $A = \frac{h}{2}(a+b)$; find h when $A = 145$, $a = 15$ and $b = 25$.

b $A = l \times w$; find w when $A = 186$ and $l = 15$.

c $V = \pi r^2 h$; find h when $V = 165.2$ and $r = 3.6$.

d $T = a + (n-1)d$; find n when $T = 260$, $a = {}^{-}15$ and $d = 11$.

e $v^2 = u^2 + as$; find s when $v = 5.5$, $u = 2.4$ and $a = 1.2$.

4 In the formula $A = 6s^2$, find the value(s) of s when $A = 150$.

5 WE11 The formula $A = \pi r^2$ is used to calculate the area of a circle. Find the radius of a circle, correct to 2 decimal places, given that the area of the circle is 328 cm^2.

6 Substitute into each of the formulas and solve the equation to find the value of the unknown. Where necessary, give your answer correct to 2 decimal places.

a $V = r^2 h$; find r when $V = 1.406\,25$ and $h = 2.5$.

b $A = \pi(R^2 - r^2)$; find R when $A = 12$ and $r = 1$.

c $V = \frac{4}{3}\pi r^3$, find r when $V = 136$.

d $E = \frac{1}{2}mv^2$; find v when $E = 254$ and $m = 23$.

e $P = I^2 R$; find I when $P = 0.54$ and $R = 1.5$.

7 WE12 Make x the subject of the formula $y = 2x + 1$.

8 Make l the subject of the formula $A = l \times b$.

9 In the formula $A = \frac{h}{2}(a+b)$:

a make a the subject of the formula

b make h the subject of the formula.

10 WE13 Make r the subject of the formula $A = \pi r^2$.

11 In the formula $E = mc^2$:

a make m the subject of the formula

b make c the subject of the formula.

Questions 12 to 14 refer to the following information.

The volume of a square-based pyramid with the side of the base, s, and the height, h, is given by the formula $V = \frac{1}{3}s^2 h$.

12 MC The side length of the base of a square-based pyramid with the height, h, and volume, V, is given by:

A $s = 3\sqrt{\frac{V}{h}}$

B $s = \sqrt{\frac{h}{3V}}$

C $s = \dfrac{V}{3h}$

D $s = \sqrt{\dfrac{3V}{h}}$

13 MC The height of a square-based pyramid with the side of the base 5 cm and the volume 75 cm^3 is:

A 8 cm
B 9 cm
C 10 cm
D 12 cm

14 MC If both the side of the base and the height are doubled the volume is:

A doubled
B increased by 4 times
C increased by 6 times
D increased by 8 times

15 In each of the following, make the subject of the formula the pronumeral indicated in brackets.

a $V = \pi r^2 \quad [r]$

b $v^2 = u^2 + as \quad [u]$

c $V = \frac{4}{3}\pi r^3 \quad [r]$

d $T = 2\pi\sqrt{\dfrac{L}{g}} \quad [L]$

e $c^2 = a^2 + b^2 \quad [a]$

Further development

16 In each of the following make the pronumeral in brackets the subject and then substitute the given value to find the value of the given pronumeral.

a $A = l^2 \quad [l] \quad A = 60$

b $V = \frac{4}{3}\pi r^3 \quad [r] \quad V = 1000$

c $F = mg - kv^2 \quad [v] \quad F = 250,\ m = 60,\ g = 10,\ k = 0.1$

d $v = u + at \quad [a] \quad v = 25,\ u = 0,\ t = 6$

e $S = \pi r(r + h) \quad [h] \quad S = 120,\ r = 2,\ \pi = 3.14$

f $T = 2\pi\sqrt{\dfrac{L}{g}} \quad [l] \quad T = 4,\ g = 9.8,\ \pi = 3.14$

g $f = \dfrac{l^2 - d^2}{4l} \quad [d] \quad f = 2,\ l = 15$

h $H = U + PV \quad [V] \quad H = 26,\ U = 4.5,\ P = 2$

i $K = \dfrac{c\alpha^2}{1 - \alpha} \quad [c] \quad K = 6.9,\ \alpha = 0.05$

j $\dfrac{H_i}{H_o} = \dfrac{v}{u} \quad [u] \quad H_i = 34,\ H_o = 4,\ v = 40$

17 The perimeter P of a rectangle of length l and width w may be found using the equation $P = 2(l + w)$.

a Find the perimeter of a rectangle of length 16 cm and width 5 cm.
b Rearrange the equation to make w the subject.
c Find the width of a rectangle that has perimeter 560 mm and length 240 mm.

18 The net force F measured in newtons (N) acting on a mass m measured in kilograms (kg) is found using the equation $F = ma$, where a is the acceleration of the mass measured in metres per second .

a Find the net force required to accelerate a 2.5 kg rock at the rate of 4 m/s^2.

b Make a the subject of the equation.

c Find the acceleration produced by a 700 N force acting on a 65 kg person.

19 The object and image positions for a lens of focal length f are related by the formula $\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$, where u is the distance of the object from the lens and v is the distance of the image from the lens.

a Make f the subject of the equation.

b Make u the subject of the equation.

c How far from the lens is the image when an object is 30 cm in front of a lens of focal length 25 cm?

5E Solution by substitution

The answer to an equation can be checked by substituting the solution back into the original equation. If the solution is correct, then the value that is substituted will satisfy the equation. For example, consider the following equation.

$$4x - 5 = 19$$
$$4x = 24$$
$$x = 6$$

Substituting $x = 6$ into $4x + 5 = 19$

Left-hand side (LHS) $= 4 \times 6 - 5$

$= 19$

$=$ Right-hand side (RHS)

By substitution it can be seen that $x = 6$ is the correct solution to this equation.

Some more difficult equations can have an approximate solution found by substituting a first guess into the equation and gradually refining the solution. This method of solution is sometimes called a 'guess and check' method.

WORKED EXAMPLE 14

Find an approximate solution to the equation $2^x = 20$ (correct to 1 decimal place).

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Worked example 14

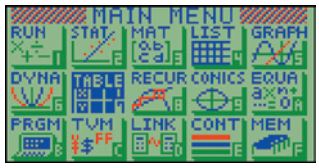
THINK	WRITE
1 Make a first guess ($x = 4$) and substitute into the equation.	Test $x = 4$ $2^4 = 16$
2 As $2^4 < 20$, make a second guess that is greater than 4 ($x = 5$).	Test $x = 5$ $2^5 = 32$
3 As $2^5 > 20$, make the next estimate between 4 and 5 ($x = 4.5$).	Test $x = 4.5$ $2^{4.5} = 22.6$
4 As $2^{4.5} > 20$, make the next estimate between 4 and 4.5 ($x = 4.3$).	Test $x = 4.3$ $2^{4.3} = 19.7$
5 As $2^{4.3} < 20$, make the next estimate between 4.3 and 4.5 ($x = 4.4$).	Test $x = 4.4$ $2^{4.5} = 21.1$
6 Since $x = 4.3$ gives a result closer to 20 than $x = 4.4$, the solution, correct to 1 decimal place, is $x = 4.3$.	Solution is $x = 4.3$.

Many such equations arise from a practical situations such as those involving investments.

WORKED EXAMPLE 15

Terry has $1000 to invest; however, he needs $1500 to purchase the electric guitar that he wants. If Terry invests his $1000 at 6% p.a., the amount in the account at any time can be found using the formula $A = 1000(1.06)^n$, where n is the number of years for which the money has been invested. Find how long it will take (correct to the nearest year) for Terry's $1000 to grow to $1500.

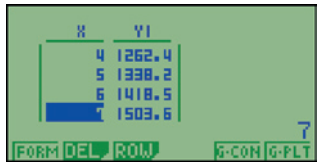
Price $1500

THINK	WRITE
Method 1: Technology-free	
1 Write the formula.	$A = 1000(1.06)^n$
2 Substitute $A = 1500$.	$1500 = 1000(1.06)^n$
3 Divide both sides by 1000.	$1.5 = (1.06)^n$
4 Make a first estimate for the solution ($n = 5$).	$(1.06)^5 = 1.34$
5 As $(1.06)^5 < 1.5$, make a second estimate greater than $n = 5$ ($n = 8$).	$(1.06)^8 = 1.59$
6 As $(1.06)^8 > 1.5$, make the next estimate between $n = 5$ and $n = 8$ ($n = 7$).	$(1.06)^7 = 1.50$
7 The solution must be $n = 7$ as $(1.06)^7 = 1.5$, correct to 2 decimal places.	It will take 7 years for the $1000 to grow to $1500 at 6% p.a
Method 2: Technology-enabled	
1 From the **MENU** select **TABLE**.	

2. For **Y1** enter the function **1000 × 1.06^X** and press EXE.

3. Press F5 **(RANG)** and then enter the settings shown on the screen opposite. These determine the start and end values and the pitch of the table. The *pitch* is the increment by which x changes from the start value.

4. Press EXE to return to the previous screen, and then press F6 **(TABL)**. You will then need to scroll down to find the value of **Y1** that is closed to 1500.



This type of question can be solved using a graphics calculator and by setting up a table of values.

REMEMBER

1. Equations such as $2^x = 10$ have no opposite operation that you can use easily. Find an approximate solution by substitution and then improve on the first estimate.
2. The first estimate is taken and substituted into the equation. A second estimate, either higher or lower than the first depending on the result of the substitution, is then taken.
3. Further estimates can then be taken by dividing the range within which we know the solution lies.
4. Most equations of this type can be solved using the table function on a graphics calculator.

EXERCISE 5E

Solution by substitution

1 Solve the equation $2^x = 100$, correct to 1 decimal place.

2 WE14 Solve the equation $1.1^x = 2$, correct to the nearest whole number.

3 Solve the equation $0.9^x = 0.5$, correct to the nearest whole number.

4 WE15 The amount of time that it will take for an investment to double when invested at 5% p.a. can be calculated using the equation $(1.05)^n = 2$. Find the value of n, correct to the nearest whole number.

5 It is anticipated that the value of a house will keep pace with inflation. Judy purchased a house in 2001 for $265 000. The future value of the house can be calculated using the formula $A = P\left(1 + \frac{r}{100}\right)^n$, where A is the future value, P is the present value, r is the inflation rate and n is the number of years. Judy wants

to know how many years it will take for the value of her property to exceed $500 000 given that the inflation rate will average 4% p.a.

a Substitute the known values into the formula to create an equation.

b Solve the equation for n, correct to the nearest whole number.

6 The value of a computer decreases at the rate of 30% p.a. A new computer purchased for $3000 can have its value after n years calculated using the formula $V = 3000(0.7)^n$, where n is the age of the computer in years. Calculate when the value of the computer will equal $500, correct to the nearest year.

7 The distance through which an object will fall in t seconds can be calculated using the formula $d = 5t^2$.

a Copy and complete the table below.

t	1	2	3	4	5	6	7	8	9	10
d										

b Calculate the length of time that it will take an object to fall 300 m, correct to the nearest second.

8 Kayla has 80 m of fencing in which to enclose a rectangular area for a garden.

a Copy and complete the table below.

Length	5	10	15	20	25	30	35
Width							
Area							

b What dimensions should the garden be if it is to enclose the maximum possible area?

c The garden is to use an existing fence for one side and use the 80 m of fencing to build the other three sides. Draw up a table to calculate the dimensions Kayla should now build the garden to maximise the area.

Further development

9 Use a guess-and-check method to find the number of years that it will take for a sum of money to double when invested at 9% p.a. compound interest with interest compounded annually.

10 Yvonne has a sum of money that she needs to double over the next 10 years. If the money is invested at compound interest with interest compounded annually, calculate the minimum interest rate required, as a percentage, correct to 1 decimal place.

11 John has invested $6000 at compound interest, which has been compounded annually. After 5 years he has $8000. Find the percentage interest rate as a percentage correct to 1 decimal place.

12 Use a guess-and-check method to solve the equation $x^3 - 8x = 85$.

13 The growth rate of a population is 15% p.a. Given that there is a population of 15 000 people in 2010, find the number of years that it will take for the population to first exceed 50 000.

14 The height of an object thrown off a cliff at any time can be found using the equation $h = 75 + 10t - 5t^2$. Find the length of time that it would take for the object to hit the ground.

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SUMMARY

Scientific notation

- Scientific notation is used to write very large or very small numbers in a shorthand way using powers of 10.
- The decimal point is moved after the first significant figure and is multiplied by the appropriate power of 10.
- For large numbers, the power of 10 is the number of places the decimal point has been moved to the left.
- For small numbers, the power of 10 is negative and is the number of places the decimal point has been moved to the right.

Substitution

- Substitution involves the replacement of a pronumeral with a numerical value in an expression.
- These expressions include linear expressions that have only powers of 1, quadratics that have a power of 2 and cubics that have a power of 3.
- Care must be taken when using a calculator to apply the power to the correct term.

Algebraic manipulation

- Algebraic terms are added or subtracted by collecting like terms.
- Algebraic terms are multiplied or divided by applying the index laws to each pronumeral separately.

First Index Law: $a^x \times a^y = a^{x+y}$

Second Index Law: $a^x \div a^y = a^{x-y}$

Third Index Law: $(a^x)^y = a^{xy}$

Equations and formulas

- After substituting into a formula, an equation will be created when you are not finding the subject of the formula.
- The equation that you may need to solve could be linear or quadratic.
- Using the same method as for solving equations, you can rearrange a formula to make another pronumeral the subject of the formula.

Solution by substitution

- Some equations have no opposite operation that allows you to easily solve the equation. These equations can have an approximate solution found using substitution.
- To solve an equation using this method, make a first estimate of the solution and substitute that estimate into the equation.
- Use the result of that substitution to make an improved estimate, and then substitute the improved estimate into the equation. Repeat this process until a solution to the desired degree of accuracy is found.

CHAPTER REVIEW

MULTIPLE CHOICE

1 A square has a side length of 5.6×10^5 cm. The area of the square in scientific notation will be:
- **A** 3.136×10^{11} cm^2
- **B** 31.36×10^{10} cm^2
- **C** 3.136×10^{25} cm^2
- **D** 31.36×10^{25} cm^2

2 $3x(2x - 4y) - 2y(4y - 6x) =$
- **A** $6x^2 - 8y^2$
- **B** $6x^2 + 8y^2$
- **C** $6x^2 - 24xy - 8y^2$
- **D** $6x^2 - 24xy + 8y^2$

3 The total surface area of a cone is given by the formula $A = \pi r(r + s)$, where r is the radius and s is the slant height of the cone. The formula with s as the subject is:
- **A** $s = \dfrac{A}{\pi r} - r$
- **B** $s = \dfrac{A - \pi r}{r}$
- **C** $s = \dfrac{A - r}{\pi r}$
- **D** $s = \dfrac{A}{\pi r} + r$

4 The total surface area of the square-based pyramid with side of the base b and the height of the triangular face h is given by $A = b^2 + 2bh$. If the total surface area of the pyramid is 64 cm and the length of the side of the base is 4 cm, the height of the triangular face is:
- **A** 6 cm
- **B** 10 cm
- **C** 20 cm
- **D** 24 cm

5 The solution to the equation $10^x = 200$ is closest to:
- **A** 2
- **B** 2.3
- **C** 2.4
- **D** 20

SHORT ANSWER

1 Write each of the following in scientific notation.
- **a** 600 000
- **b** 0.000 000 000 2
- **c** 78 920 000 000 000
- **d** 0.001 25
- **e** 0.000 004 589
- **f** 124 589 000 000 000

2 Write each of the following in scientific notation, correct to 3 significant figures.
- **a** 12 589
- **b** 0.000 125 478 624
- **c** 0.032 143 68
- **d** 586 460 484
- **e** 12 447.151 48
- **f** 0.000 000 051 851 58

3 Write each of the following as a decimal number.
- **a** 2.5×10^2
- **b** 3.87×10^4
- **c** 9.8504×10^7
- **d** 2.89×10^{-1}
- **e** 3.6702×10^{-7}
- **f** 1.1×10^{-3}

4 Complete each of the following.
- **a** 2.5×10^5 m = ___ mm
- **b** 2.8×10^8 g = ___ kg
- **c** 3.43×10^4 kL = ___ L
- **d** 1.45×10^6 m = ___ km
- **e** 4.243×10^7 t = ___ kg
- **f** 1.3×10^8 mL = ___ L

5 Find the value of $V = A \times h$, when $A = 54$ and $h = 3$.

6 Find the value of $S = ut + \frac{1}{2}at^2$, when $u = 4.1$, $t = 6.2$ and $a = 0.6$.

7 Find the value of $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$, when $x_1 = -2$, $y_1 = -7$, $x_2 = 3$ and $y_2 = 5$.

8 Find the value of each of the following giving your answer, where necessary, correct to 2 decimal places.
- **a** $A = \pi r(r + s)$, when $r = 3.9$ and $s = 7.2$
- **b** $C = \frac{5}{9}(F - 32)$, when $F = 100$
- **c** $S = \dfrac{a}{1 - r}$, when $a = 12$ and $r = -0.4$
- **d** $y = \sqrt{r^2 - x^2}$, when $r = 10$ and $x = 6$

9 Simplify each of the following by collecting like terms.
- **a** $m + m + m + m + m$
- **b** $7q + 9q$
- **c** $5p + 8p - p$
- **d** $23t - 22t$
- **e** $4m + 6n - 2n$
- **f** $7x + 4 - 3x - 9$

g $11k - 6l + 4l - 8k$
h $5x^2 + 20x + 3x^2 - 6x$
i $4ab + 7a - 2b - 3ba$

10 Simplify each of the following.
a $4a^4 \times 7a^5$
b $5b \times 9b$
c $3g^2h^5 \times 7g^2h^3$
d $12m^5n^6 \times mn$
e $42x^6 \div 7x^4$
f $32r^5s^4 \div 4r^5s$
g $\dfrac{28q^2}{4q}$
h $(3p^2q^4)^3$
i $(8m^2)^2 \times m \div 16m^3$

11 Expand each of the following.
a $2(a + 9)$
b $p(2p - 4)$
c $^-x^2(3x^3 - 1)$
d $4m^5(3m^2 - 2n)$
e $^-4xy(4 - y)$
f $6a^2b^3(2a^3 - 4b^2)$

12 Expand and simplify each of the following.
a $2(m + 8) + 6(m + 4)$
b $3p(p - 2) + p(3 - p)$
c $7(2x - 4) - 3(x + 8)$
d $3z(y - 2z) + 4y(2y + z)$
e $4pq(p - q) - 2p(pq - 4)$

13 In the formula $P = 2l + 2b$ find l, when $P = 78$ and $b = 24$.

14 The formula $C = 2\pi r$ is used to find the circumference of a circle given the radius. Find the radius of a circle with a circumference of 136 m. Give your answer correct to 1 decimal place.

15 In the formula $A = 6s^2$, find s when $A = 216$.

16 The volume of a square-based pyramid can be found using the formula $V = \frac{1}{3}s^2h$, where s is the side length of the square base and h is the height of the pyramid. Find the side length of a square-based pyramid with a volume of 108.864 cm^3 and a height of 6.3 cm.

17 Use the method of substitution to solve the following equations, correct to 1 decimal place.
a $5^x = 100$
b $(1.2)^x = 2$
c $(0.75)^x = 0.25$

18 The amount to which $10 000 will grow when invested at 9% p.a. can be found using the formula $A = 10\,000\,(1.09)^n$, where n is the number of years of the investment. Use the formula to find the amount of time that it will take for $10 000 to grow into $20 000, correct to the nearest year.

19 A car depreciates at a rate of 20% p.a. The amount of time that it takes for the car to halve in value can be found by solving the equation $(0.8)^n = 0.5$, where n is the age of the car. Find the length of time it takes for a car to halve in value, correct to the nearest year.

EXTENDED RESPONSE

1 The volume of a cylinder can be found using the formula $V = \pi r^2 h$. The surface area of a cylinder can be found using the formula $SA = 2\pi r^2 + 2\pi rh$.

- **a** Find the volume of a cylinder with a radius of 4.2 cm and a height of 5.5 cm. (Give your answer correct to 1 decimal place.)
- **b** Find the height of a cylinder with a volume of 705 cm^3 and a radius of 5.2 cm. (Give your answer correct to 1 decimal place.)
- **c** Find the radius of a cylinder with a volume of 939.4 cm^3 and a height of 7.3 cm (correct to 1 decimal place).
- **d** Rewrite the formula for surface area to make h the subject.

2 The time taken for an investment to double in value when invested at 7.5% p.a. can be found by solving the equation $(1.075)^n = 2$.

- **a** Use the method of substitution to find the solution to this equation, correct to the nearest whole number.
- **b** Write an equation that can be used to find the amount of time that it will take for the value of an item to halve in value if it depreciates at 15% p.a.
- **c** Solve this equation, correct to 1 decimal place.

eBookplus ACTIVITIES

Are you ready?

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- SkillSHEET 5.2 (doc-1349): Substitution into a formula.
- SkillSHEET 5.3 (doc-1350): Simplifying like terms.
- SkillSHEET 5.4 (doc-1352): Multiplication using indices.
- SkillSHEET 5.5 (doc-1353): Division using indices.
- SkillSHEET 5.6 (doc-1355): Raising a power to a power.
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5A Scientific notation

Tutorial
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5B Substitution

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5C Algebraic manipulation

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5E Solution by substitution

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